Models for Retrieval and Browsing

- Fuzzy Set, Extended Boolean, Generalized Vector Space Models

Berlin Chen 2003

Reference:

1. Modern Information Retrieval, chapter 2

Outline

- Alternative Set Theoretic Models
  - Fuzzy Set Model (Fuzzy Information Retrieval)
  - Extended Boolean Model

- Alternative Algebraic Models
  - Generalized Vector Space Model

Fuzzy Set Model

- Fuzzy Set Theory
  - Framework for representing classes whose boundaries are not well defined
  - Key idea is to introduce the notion of a *degree of membership* associated with the elements of a set
  - This degree of membership varies from 0 to 1 and allows modeling the notion of *marginal* membership
  - Thus, membership is now a gradual instead of abrupt (as conventional Boolean logic)

Fuzzy Set Model

- Definition
  - A fuzzy subset A of a universal of discourse U is characterized by a membership function $\mu_A: U \rightarrow [0,1]$
    - which associates with each element u of U a number $\mu_A(u)$ in the interval [0,1]
  - Let A and B be two fuzzy subsets of U. Also, let $\overline{A}$ be the complement of A. Then,
    - Complement $\mu_{\overline{A}}(u) = 1 - \mu_A(u)$
    - Union $\mu_{A \cup B}(u) = \max(\mu_A(u), \mu_B(u))$
    - intersection $\mu_{A \cap B}(u) = \min(\mu_A(u), \mu_B(u))$

Fuzzy Set Model

- Fuzzy information retrieval
  - Fuzzy sets are modeled based on a thesaurus
  - This thesaurus is constructed by a term-term correlation matrix
    - $\vec{c}$: a term-term correlation matrix
    - $c_{i,l}$: a normalized correlation factor for terms k_i and k_l

$$c_{i,l} = \frac{n_{i,l}}{n_i + n_l - n_{i,l}}$$

n_i : no of docs that contain k_i
$n_{i,l}$: no of docs that contain both k_i and k_l

    - We now have the notion of proximity among index terms
  - The union and intersection operations are modified here
    - Union: algebraic sum (instead of max)
    - Intersection: algebraic product (instead of min)

Fuzzy Set Model

– The degree of membership between a doc d_j and an index term k_i

$$u_{i,j} = 1 - \prod_{k_l \in d_j} \left(1 - c_{i,j}\right)$$

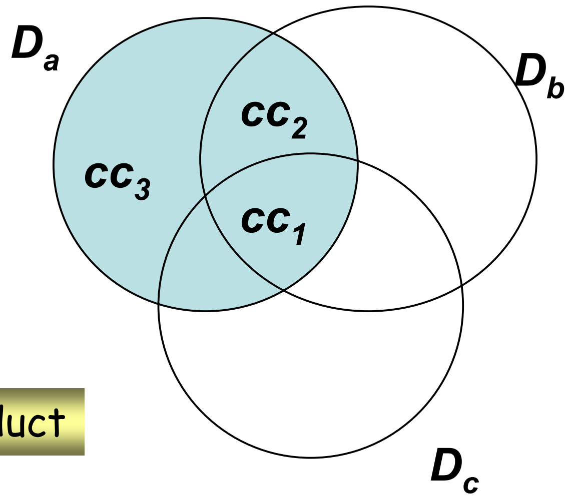
- Computes an **algebraic sum** (instead of max function) over all terms in the doc d_j
  – Implemented as the complement of a negative algebraic product (why?)
- A doc d_j belongs to the fuzzy set associated to the term k_i if its own terms are related to k_i
- If there is at least one index term k_l of d_j which is strongly related to the index ($c_{i,l} \sim 1$) then $\mu_{i,j} \sim 1$
  – k_i is a good fuzzy index for doc d_j
  – *And vice versa*

Fuzzy Set Model

- Example:
  - Query $q = k_a \wedge (k_b \vee \neg k_c)$
    $\vec{q}_{dnf} = (k_a \wedge k_b \wedge k_c) \vee (k_a \wedge k_b \wedge \neg k_c) \vee (k_a \wedge \neg k_b \wedge \neg k_c)$
    $= cc_1 + cc_2 + cc_3$

disjunctive normal form

  - D_a is the fuzzy set of docs associated to the term k_a
  - Degree of membership



$$\mu_{q,j} = \mu_{cc_1+cc_2+cc_3,j}$$

algebraic sum

$$= 1 - \prod_{i=1}^{3} (1 - \mu_{cc_i,j})$$

negative algebraic product

$$= 1 - (1 - \underbrace{\mu_{a,j}\mu_{b,j}\mu_{c,j}}_{cc_1}) \times (1 - \underbrace{\mu_{a,j}\mu_{b,j}(1-\mu_{c,j})}_{cc_2}) \times (1 - \underbrace{\mu_{a,j}(1-\mu_{b,j})(1-\mu_{c,j})}_{cc_3})$$

algebraic product

Fuzzy Set Model

- Fuzzy IR models have been discussed mainly in the literature associated with fuzzy theory
- Experiments with standard test collections are not available

Extended Boolean Model

Salton et al., 1983

- Motive
  - Extend the Boolean model with the functionality of partial matching and term weighting
    - E.g.: in Boolean model, for the qery $q=k_x \wedge k_y$, a doc contains either k_x or k_y is as irrelevant as another doc which contains neither of them
  - Combine Boolean query formulations with characteristics of the vector model
    - Term weighting
    - Algebraic distances for similarity measures

} a ranking can be obtained

Extended Boolean Model

- Term weighting
  - The weight for the term k_x in a doc d_j is

$$w_{x,j} = tf_{x,j} \times \frac{idf_x}{\max_i idf_i}$$

Normalized *idf*

    - $w_{x,j}$ is normalized to lay between 0 and 1

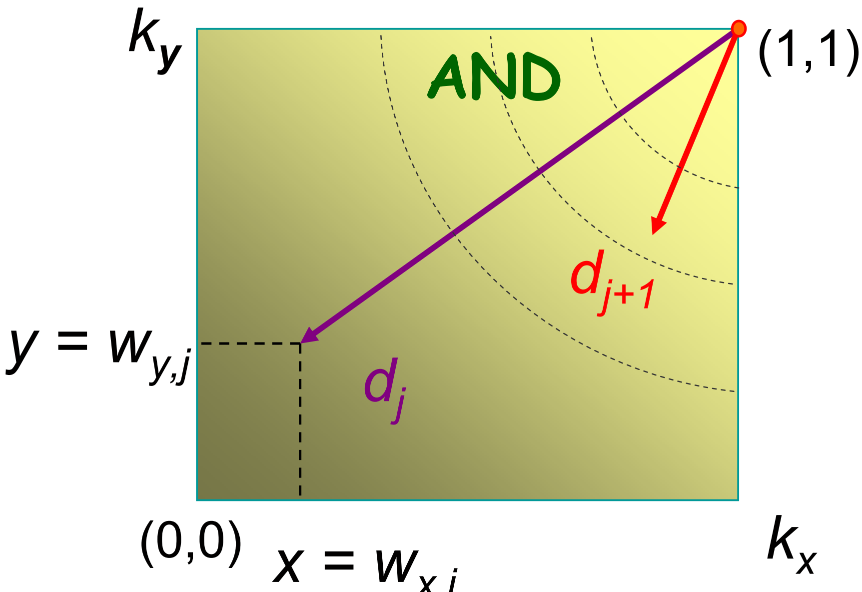
- Assume two index terms k_x and k_y were used
  - Let x denote the weight $w_{x,j}$ of term k_x on doc d_j
  - Let y denote the weight $w_{y,j}$ of term k_y on doc d_j
  - The doc vector $\vec{d}_j = (w_{x,j}, w_{y,j})$ is represented as $d_j = (x, y)$
  - Queries and docs can be plotted in a two-dimensional map

Extended Boolean Model

- If the query is $q=k_x \wedge k_y$ (conjunctive query)
  - -The docs near the point (1,1) are preferred
  - -The similarity measure is defined as

$$sim(q_{and}, d) = 1 - \sqrt{\frac{(1-x)^2 + (1-y)^2}{2}}$$

2-norm model

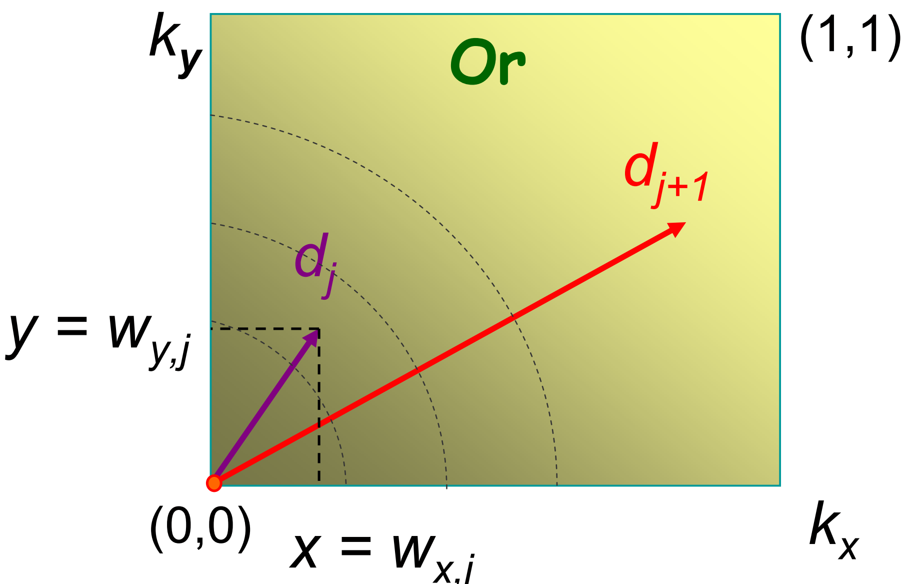


Extended Boolean Model

- If the query is $q=k_x \vee k_y$ (disjunctive query)
  - -The docs far from the point (0,0) are preferred
  - -The *similarity measure* is defined as

$$sim(q_{or}, d) = \sqrt{\frac{x^2 + y^2}{2}}$$

2-norm model



Extended Boolean Model

- Generalization
  - *t* index terms are used → *t*-dimensional space
  - *p*-norm model, $1 \leq p \leq \infty$

$$q_{and} = k_1 \wedge^p k_2 \wedge^p \ldots \wedge^p k_m \Rightarrow sim(q_{and}, d) = 1 - \left(\frac{(1-x_1)^p + (1-x_2)^p + \ldots + (1-x_m)^p}{m} \right)^{\frac{1}{p}}$$

$$q_{or} = k_1 \vee^p k_2 \vee^p \ldots v^p k_m \Rightarrow sim(q_{or}, d) = \left(\frac{x_1^{\ p} + x_2^{\ p} + \ldots + x_m^{\ p}}{m} \right)^{\frac{1}{p}}$$

  - Some interesting properties
    - *p=1* $\Rightarrow$ $sim(q_{and}, d) = sim(q_{or}, d) = \frac{x_1 + x_2 + \ldots + x_m}{m}$
    - *p=*∞ $\Rightarrow$ $sim(q_{and}, d) = \min(x_i)$

      $sim(q_{or}, d) = \max(x_i)$

Extended Boolean Model

- Example query 1: $q = \left(k_1 \wedge^p k_2\right) \vee^p k_3$
  - Processed by grouping the operators in a predefined order

$$sim(q,d) = \left(\frac{\left(1 - \left(\frac{(1-x_1)^p + (1-x_2)^p}{2}\right)^{\frac{1}{p}}\right)^p + x_3^p}{2} \right)^{\frac{1}{p}}$$

- Example query 2: $q = \left(k_1 \vee^2 k_2\right) \wedge^\infty k_3$
  - Combination of different algebraic distances

$$sim(q,d) = \min\left(\left(\frac{x_1^2 + x_2^2}{2}\right)^{\frac{1}{2}}, x_3 \right)$$

Extended Boolean Model

- Advantages $q = (k_1 \wedge^p k_2) \vee^p k_3$
  - A hybrid model including properties of both the set theoretic models and the algebraic models
    - Relax the Boolean algebra by interpreting Boolean operations in terms of algebraic distances
- Disadvantages
  - Distributive operation does not hold for ranking computation
    - E.g.: $q_1 = (k_1 \wedge k_2) \vee k_3, q_2 = (k_1 \vee k_3) \wedge (k_2 \vee k_3)$

$$sim(q_1, d) \neq sim(q_2, d)$$

  - Assumes mutual independence of index terms

Generalized Vector Model

Wong et al., 1985

- Premise
  - Classic models enforce *independence* of index terms
  - For the **Vector model**
    - Set of term vectors $\{\vec{k_1}, \vec{k_1}, \ldots, \vec{k_t}\}$ are linearly independent and form a basis for the subspace of interest
    - Frequently, it means *pairwise orthogonality*
      - $\forall i,j \Rightarrow \vec{k_i} \bullet \vec{k_j} = 0$ (in a more restrictive sense)

- Wong et al. proposed an interpretation
  - The index term vectors are *linearly independent*, but *not pairwise orthogonal*
    - Generalized Vector Model

Generalized Vector Model

- **Key idea** of Generalized Vector Model
  - Index term vectors form the basis of the space are not orthogonal and are represented in terms of smaller components (minterms)

- **Notations**
  - $\{k_1, k_2, ..., k_t\}$: the set of all terms
  - $w_{i,j}$: the weight associated with $[k_i, d_j]$
  - **Minterms**:binary indicators (0 or 1) of all patterns of occurrence of terms within documents
    - Each represent one kind of co-occurrence of index terms in a specific document

Generalized Vector Model

- Representations of **minterms**

$m_1=(0,0,\ldots,0)$
$m_2=(1,0,\ldots,0)$
$m_3=(0,1,\ldots,0)$
$m_4=(1,1,\ldots,0)$
$m_5=(0,0,1,..,0)$
…
$m_{2^t}=(1,1,1,..,1)$

2^t minterms

Points to the docs where only index terms k_1 and k_2 co-occur and the other index terms disappear

Point to the docs containing all the index terms

$\vec{m}_1=(1,0,0,0,0,\ldots,0)$
$\vec{m}_2=(0,1,0,0,0,\ldots,0)$
$\vec{m}_3=(0,0,1,0,0,\ldots,0)$
$\vec{m}_4=(0,0,0,1,0,\ldots,0)$
$\vec{m}_5=(0,0,0,0,1,\ldots,0)$
…
$\vec{m}_{2^t}=(0,0,0,0,0,\ldots,1)$

2^t minterm vectors

Pairwise orthogonal vectors $\vec{m}_i$ associated with minterms m_i as the **basis** for the ***generalized vector space***

Generalized Vector Model

- Minterm vectors are *pairwise orthogonal*. But, this does not mean that the index terms are independent
  - Each minterm specifies a kind of dependence among index terms

Generalized Vector Model

- The vector associated with the term k_i is represented by **summing** up all minterms containing it and **normalizing**

$$\vec{k}_i = \frac{\sum_{\forall r, g_i(m_r)=1} c_{i,r} \vec{m}_r}{\sqrt{\sum_{\forall r, g_i(m_r)=1} c_{i,r}^2}}$$

$$c_{i,r} = \sum_{d_j \mid g_l(\vec{d}_j) = g_l(m_r), \text{ for all } l} w_{i,j}$$

•The weight associated with the pair [k_i, m_r] sums up the weights of the term k_i in all the docs which have a term occurrence pattern given by m_r.

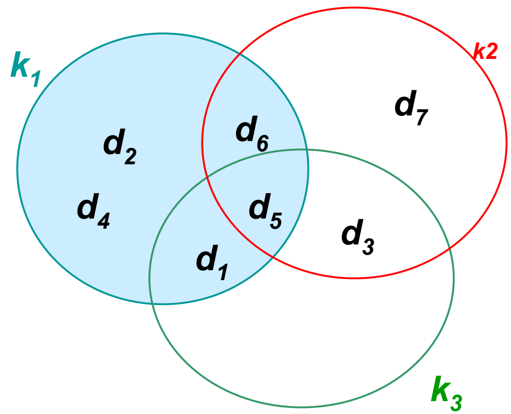
•Notice that for a collection of size N, only N minterms affect the ranking (and not

All the docs whose term co-occurrence relation (pattern) can be represented as (exactly coincide with that of) minterm m_r

Generalized Vector Model

- **Example** (a system with three index terms)

minterm	k_1	k_2	k_3
m_1	0	0	0
m_2	1	0	0
m_3	0	1	0
m_4	1	1	0
m_5	0	0	1
m_6	1	0	1
m_7	0	1	1
m_8	1	1	1



$$\vec{k}_1 = \frac{c_{1,2}\vec{m}_2 + c_{1,4}\vec{m}_4 + c_{1,6}\vec{m}_6 + c_{1,8}\vec{m}_8}{\sqrt{c_{1,2}^2 + c_{1,4}^2 + c_{1,6}^2 + c_{1,8}^2}}$$

$$\vec{k}_2 = \frac{c_{2,3}\vec{m}_3 + c_{2,4}\vec{m}_4 + c_{2,7}\vec{m}_7 + c_{2,8}\vec{m}_8}{\sqrt{c_{2,3}^2 + c_{2,4}^2 + c_{2,7}^2 + c_{2,8}^2}}$$

$$\vec{k}_3 = \frac{c_{3,5}\vec{m}_5 + c_{3,6}\vec{m}_6 + c_{3,7}\vec{m}_7 + c_{3,8}\vec{m}_8}{\sqrt{c_{3,5}^2 + c_{3,6}^2 + c_{3,7}^2 + c_{3,8}^2}}$$

	k_1	k_2	k_3	minterm
d_1	2	0	1	m_6
d_2	1	0	0	m_2
d_3	0	1	3	m_7
d_4	2	0	0	m_2
d_5	1	2	4	m_8
d_6	1	2	0	m_4
d_7	0	5	0	m_3
q	1	2	3	

$$c_{1,2} = w_{1.2} + w_{1.4} = 1 + 2 = 3$$
$$c_{1,4} = w_{1.6} = 1$$
$$c_{1,6} = w_{1,1} = 2$$
$$c_{1,8} = w_{1,5} = 1$$

$$\vec{k}_1 = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{3^2 + 1^2 + 2^2 + 1^2}}$$

$$c_{2,3} = w_{2,7} = 5$$
$$c_{2,4} = w_{2,6} = 2$$
$$c_{2,7} = w_{2,3} = 1$$
$$c_{2,8} = w_{2,5} = 2$$

$$\vec{k}_2 = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{5^2 + 2^2 + 1^2 + 2^2}}$$

$$c_{3,5} = 0$$
$$c_{3,6} = w_{3,1} = 1$$
$$c_{3,7} = w_{3,3} = 3$$
$$c_{3,8} = w_{3,5} = 4$$

$$\vec{k}_3 = \frac{0\vec{m}_5 + 1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{0^2 + 1^2 + 3^2 + 4^2}}$$

Generalized Vector Model

- **Example**: Ranking

$$\vec{k}_1 = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{3^2+1^2+2^2+1^2}} = \frac{3\vec{m}_2 + 1\vec{m}_4 + 2\vec{m}_6 + 1\vec{m}_8}{\sqrt{15}}$$

$$\vec{k}_2 = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{5^2+2^2+1^2+2^2}} = \frac{5\vec{m}_3 + 2\vec{m}_4 + 1\vec{m}_7 + 2\vec{m}_8}{\sqrt{34}}$$

$$\vec{k}_3 = \frac{0\vec{m}_5 + 1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{0^2+1^2+3^2+4^2}} = \frac{1\vec{m}_6 + 3\vec{m}_7 + 4\vec{m}_8}{\sqrt{26}}$$

$$\vec{d}_1 = 2\vec{k}_1 + 1\vec{k}_3$$

$$= \underbrace{\frac{2\cdot 3}{\sqrt{15}}}_{s_{d1,2}}\vec{m}_2 + \underbrace{\frac{2\cdot 1}{\sqrt{15}}}_{s_{d1,4}}\vec{m}_4 + \underbrace{\left(\frac{2\cdot 2}{\sqrt{15}} + \frac{1\cdot 1}{\sqrt{26}}\right)}_{s_{d1,6}}\vec{m}_6 + \underbrace{\frac{1\cdot 3}{\sqrt{26}}}_{s_{d1,7}}\vec{m}_7 + \underbrace{\left(\frac{2\cdot 1}{\sqrt{15}} + \frac{1\cdot 4}{\sqrt{26}}\right)}_{s_{d1,8}}\vec{m}_8$$

$$\vec{q} = 1\vec{k}_1 + 2\vec{k}_2 + 3\vec{k}_3$$

$$= \underbrace{\frac{1\cdot 3}{\sqrt{15}}}_{s_{q,2}}\vec{m}_2 + \underbrace{\frac{2\cdot 5}{\sqrt{34}}}_{s_{q,3}}\vec{m}_3 + \underbrace{\left(\frac{1\cdot 1}{\sqrt{15}} + \frac{2\cdot 2}{\sqrt{34}}\right)}_{s_{q,4}}\vec{m}_4 + \underbrace{\left(\frac{1\cdot 2}{\sqrt{15}} + \frac{3\cdot 1}{\sqrt{26}}\right)}_{s_{q,6}}\vec{m}_6 + \underbrace{\left(\frac{2\cdot 1}{\sqrt{34}} + \frac{3\cdot 3}{\sqrt{26}}\right)}_{s_{q,7}}\vec{m}_7 + \underbrace{\left(\frac{1\cdot 1}{\sqrt{15}} + \frac{2\cdot 2}{\sqrt{34}} + \frac{3\cdot 4}{\sqrt{26}}\right)}_{s_{q,8}}\vec{m}_8$$

$$sim(q,d) = \text{consine}(q,d) = \frac{\sum_{s_{q,i}\neq 0 \wedge s_{d,i}\neq 0} s_{q,i}\cdot s_{d,i}}{\sqrt{\sum_i s_{q,i}^2}\sqrt{\sum_i s_{d,i}^2}}$$

$$sim(q,d_1) = \frac{s_{q,2}s_{d1,2} + s_{q,4}s_{d1,4} + s_{q,6}s_{d1,6} + s_{q,7}s_{d1,7} + s_{q,8}s_{d1,8}}{\sqrt{s_{q,2}^2 + s_{q,3}^2 + s_{q,4}^2 + s_{q,6}^2 + s_{q,7}^2 + s_{q,8}^2}\sqrt{s_{d1,2}^2 + s_{d1,4}^2 + s_{d1,6}^2 + s_{d1,7}^2 + s_{d1,8}^2}}$$

Generalized Vector Model

- Term Correlation
  - The degree of correlation between the terms k_i and k_j can now be computed as

$$\vec{k}_i \bullet \vec{k}_j = \sum_{\forall r | g_i(m_r)=1 \wedge g_j(m_r)=1} c_{i,r} \times c_{j,r}$$

    - Do not need to be normalized? (because we have done it before!)

Generalized Vector Model

- Advantages
  - Model considers correlations among index terms
  - Model does introduce interesting new ideas
- Disadvantages
  - Not clear in which situations it is superior to the standard Vector model
  - Computation costs are higher