Linear Algebra
Homework 3
Due date: 2016/01/08

Note: You have to answer the questions with supporting explanations if needed. The computations have to be accomplished with paper and pencil.

1. If \( T(x_1, x_2, x_3) = (x_1 + 3x_2, x_1 - 3x_2), \) then
   (i) Find the domain and codomain of \( T; \) (ii) Find the image of \( x = (2, -2, 5) \) under \( T. \)

2. Use matrix multiplication to find the reflection of \((3, 2)\) about
   (i) the \(x\)-axis; (ii) the \(y\)-axis; (iii) the line \(y=x.\)

3. Describe the geometric effect of multiplying a vector \(x\) by a matrix \(A.\)

   (i) \(A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix};\) (ii) \(A = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}\)

4. Let \( T_1(x_1, x_2, x_3) = (x_1 - x_2, x_1 + 3x_2, 0) \) and \( T_2(x_1, x_2, x_3) = (2x_2, x_1 - 5x_2, x_1). \)
   (i) Find the standard matrices for \( T_2 \circ T_1 \) and \( T_1 \circ T_2. \)
   (ii) Use the matrices obtained in part (i) to find the formulas for \( T_2(T_1(x_1, x_2, x_3)) \) and \( T_1(T_2(x_1, x_2, x_3)). \)

5. Find the standard matrix for the following matrix operators.
   (i) \( T: R^3 \rightarrow R^3 \) reflects a vector about the \(x z\)-plane and then contracts that vector by a factor of \(1/2.\)
   (ii) \( T: R^3 \rightarrow R^3 \) projects a vector orthogonally onto the \(xz\)-plane and then projects that vector orthogonally onto the \(xy\)-plane

6. Given a matrix \(A = \begin{bmatrix} 2 & 0 \\ 2 & -1 \end{bmatrix}\)
   (i) express \(A\) as a product of elementary matrices, and then describe the effect on \(R^2\) of multiplication by \(A\) in terms of compression, expansions, reflections and shears.
   (ii) Sketch the image of the rectangle with vertices \((0, 0), (1, 0), (1, 2)\) and \((0, 2)\) under the transformation by \(A.\)
\[
A = \begin{bmatrix}
2 & 0 \\
2 & -1
\end{bmatrix}
= \begin{bmatrix}
2 & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & -1
\end{bmatrix}
\]