Linear Algebra

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Linear Algebra (LA)

• Linear algebra is a branch of mathematics and continues to figure prominently in computer science and electrical engineering
  – Computation, geometry, theory, practical applications, to name just a few

• Simply put, linear algebra is the study of vectors, matrices, vector spaces and linear transformations

\[
\begin{align*}
    a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
    a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
    \vdots & \vdots & \vdots & \vdots \\
    a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \\
\end{align*}
\]

\[
\begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\
    a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\
    \vdots & \vdots & \cdots & \vdots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \\
\end{bmatrix}
\]
Main Objectives

• Develop the definitions, concepts and theories associated with linear algebra
  – Fundamentals: vectors operations, matrices operations, determinants, Euclidean vector spaces, linear systems, etc.
  – Advanced topics: matrix diagonalization, matrix factorization, linear transforms, numerical methods, practical applications, etc.

• Learn to make effective use of linear algebra in dealing with practical issues of interest
  – E.g., multimedia (text, speech, music and image) processing

Information Retrieval (IR): Measuring Degree of Similarity between Query and Documents

\[ q = 0u_1 + 0u_2 + 3u_3 \]
\[ d_1 = 2u_1 + 4u_2 + 5u_3 \]
\[ d_2 = 3u_1 + 7u_2 + 7u_3 \]

\[ \text{sim}(d_j, q) = \cosine \ (\Theta) = \frac{\vec{d}_j \bullet \vec{q}}{|\vec{d}_j| \times |\vec{q}|} \]
\[ = \frac{\sum_{i=1}^{m} w_{i,j} \times w_{i,q}}{\sqrt{\sum_{i=1}^{m} w_{i,j}^2} \times \sqrt{\sum_{i=1}^{m} w_{i,q}^2}} \]

• Start with a matrix describing the intra- and Inter-document statistics between all terms and all documents
• Singular value decomposition (SVD) is then performed on the matrix to project all term and document vectors onto a reduced latent topical space

In the context of information retrieval (IR), matching between queries and documents can be carried out in this topical space

\[ \|A\| = \sum \sum \sigma_i \Rightarrow \|A\| = \sigma_1 + \sigma_2 + ... + \sigma_r \]
LA for Representation Learning

Image to Text

Word Prediction
Textbook & Course Website


  - Website
    

- Course Website
  
  http://berlin.csie.ntnu.edu.tw/Courses/LinearAlgebra/2014F-LA_Main.htm
Reference Books

  – Website
    http://www.laylinalgebra.com/

  – Website
    http://www.athenasc.com/probbook.html
## Tentative Topic List

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Grading (*Tentatively!*)

- Midterm and Final: 45%
- Quizzes (≥ 5 times) and Homework: 45%
- Attendance/Other: 10%

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